Tessellation Patterns from a Simply Decorated Triangle

David A. Reimann
Department of Mathematics and Computer Science
Albion College
Albion, Michigan, 49224, USA
E-mail: dreimann@albion.edu

Abstract

Geometric tilings have been a key element of architectural design throughout history. This paper describes the construction of patterns using a decorated triangle and gives several surprising examples demonstrating the use of this motif. Even with this very simple design element, a wealth of interesting patterns is possible. Such patterns can be useful in very large field architectural tilings.

1 Introduction

Geometric tilings have been a key element of architectural design throughout history. One of the most simple tessellations of the plane is the common triangular lattice, where six equilateral triangles meet at every vertex. In such a tessellation, the underlying polygons can be thought of as tiles. These tiles can also be decorated with a simple motif to produce more intricate patterns. M.C. Escher was a master at decorating tiles with people and animals, breathing life into otherwise cold geometric structures.

Even simple motifs can enhance a tiling's visual interest. For example, Truchet [1] explored the patterns obtainable from a single square tile that was bisected along a diagonal between opposite vertices. Smith [2] published an article containing a translation of Truchet's original paper with some commentary and new ideas including the use of a random tiling rather than a structured pattern. Smith also included a variant of the Truchet tile that replaced the triangular segmentation with two quarter-circle arcs, resulting in a tiling that is comprised of an aesthetically pleasing, meandering set of mostly closed curves. Pickover [3] proposed using randomly placed Truchet tiles as a way to visually detect patterns in binary data, noting "the eye perceives no particular trends in the design." However, he does note small features such as circles and dumbbells. Browne [4] noted a variety of interesting shapes are possible. The author [5] described how these simple shapes can be combined to define a font that can be used to embed textual information in the patterns.

A challenge in decorating polygonal tiles with arcs arises when the underlying polygon contains an odd number of sides. Browne [6] investigated filled patterns on regular polygons regions defined by arcs connecting midpoints of polygon sides. Browne uses a motif containing a bifurcation to decorate a triangle. The author's previous work [7] investigated motifs on regular polygons where each side was subdivided into an equal number of segments and connected using Bézier curves. The author's design uses two components to decorate an equilateral triangle having side length L. The first is a circular arc of radius $a = L/2$ that connects the midpoints of two sides and the second is a short line segment from the midpoint of the remaining side in the direction of the triangle's center. This tile decoration is shown in Figure 1. This paper describes the construction of patterns using a decorated triangle as motif and gives several surprising examples demonstrating the use of this pattern.

2 Methods and Results

There are six possible orientations in which a tile as decorated in Figure 1 can be placed. The arc is centered at one of the three vertices, giving three possible orientations of the triangle. Because the triangle has two

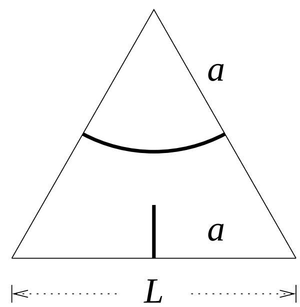

Figure 1: *The basic decorated equilateral triangle having side length L. The motif is comprised of two components. The first component of the motif is an arc of radius $a = L/2$ having endpoints that are at the midpoints of two sides. The second is a short line segment that starts at the midpoint of the third side and extends towards the center of the triangle.*

orientations in the underlying tessellation, this results in six possible orientations. An example showing a random placement of tiles is given in Figure 2. A variety of shapes is present, including circles, short line segments, and longer linear features. Because the decorations of the tile are close to the line segments that would connect the midpoints of the triangle's sides to the center, the pattern is similar to the dual of the triangular tessellation (the hexagonal tessellation).

Some striking regular patterns are also possible using this simply decorated tile. Figure 3 shows four repetitive patterns that exhibit translational symmetry and four patterns that are constructed on a hexagonal lattice. Figure 4 shows an example of a frieze pattern. Figure 5 illustrates how this motif can be used to embed text in the pattern.

3 Discussion

Even with this very simple design element, a wealth of interesting tessellated patterns is possible. One reason for the visual appeal of the patterns is the arcs of adjacent tiles are not only continuous, but also have a continuous first derivative resulting in a visually smooth transition regardless of tile orientation. In addition, the meandering paths created are roughly equally spaced, providing a relatively uniform filling of the plane. The tension present between the both local similarity and positional regularity and the irregularity of randomly generated curves provides excitement and movement not present in the underlying triangular or hexagonal tessellations. Symmetric patterns are easy to create and can be mixed with random tile orientations to create patterns with consonance and dissonance. These patterns can be useful in very large field architectural tilings.

4 Acknowledgments

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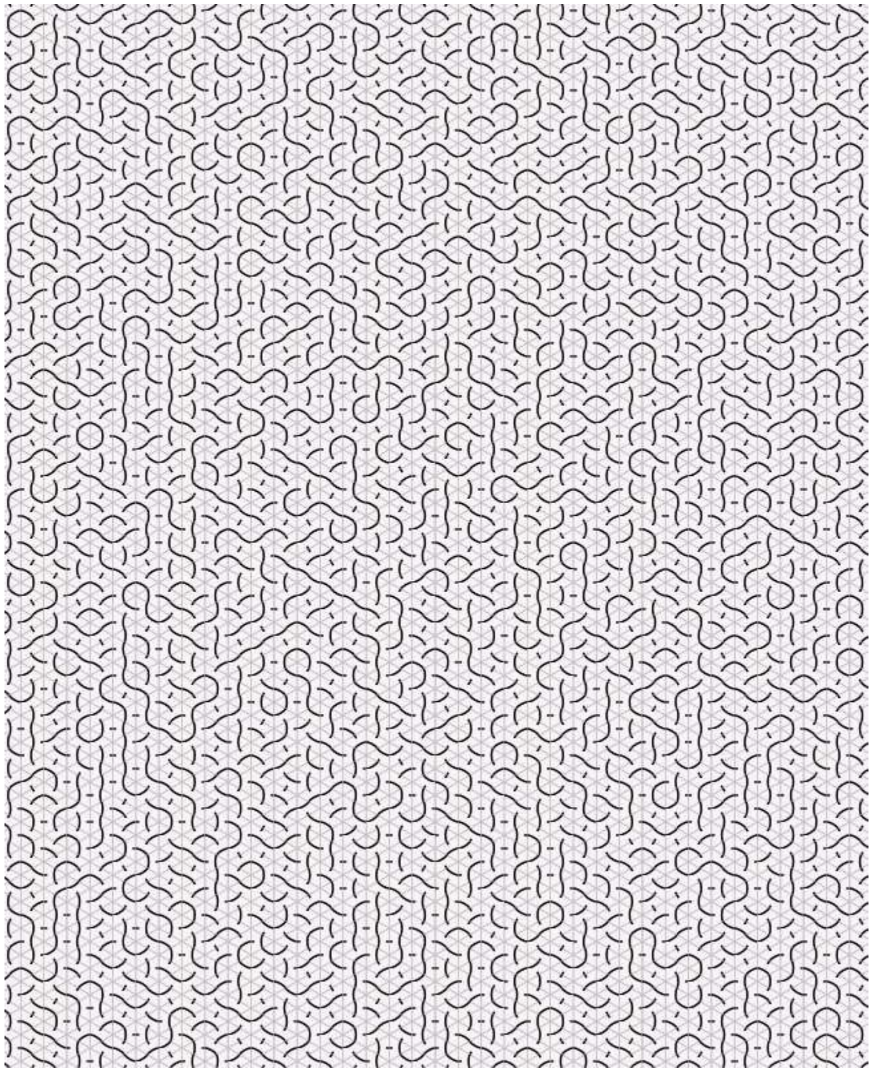

Figure 2: *A field of randomly placed tiles. The triangular lattice is shown in light gray. Note the variety and complexity of shapes created by the simple motif.*

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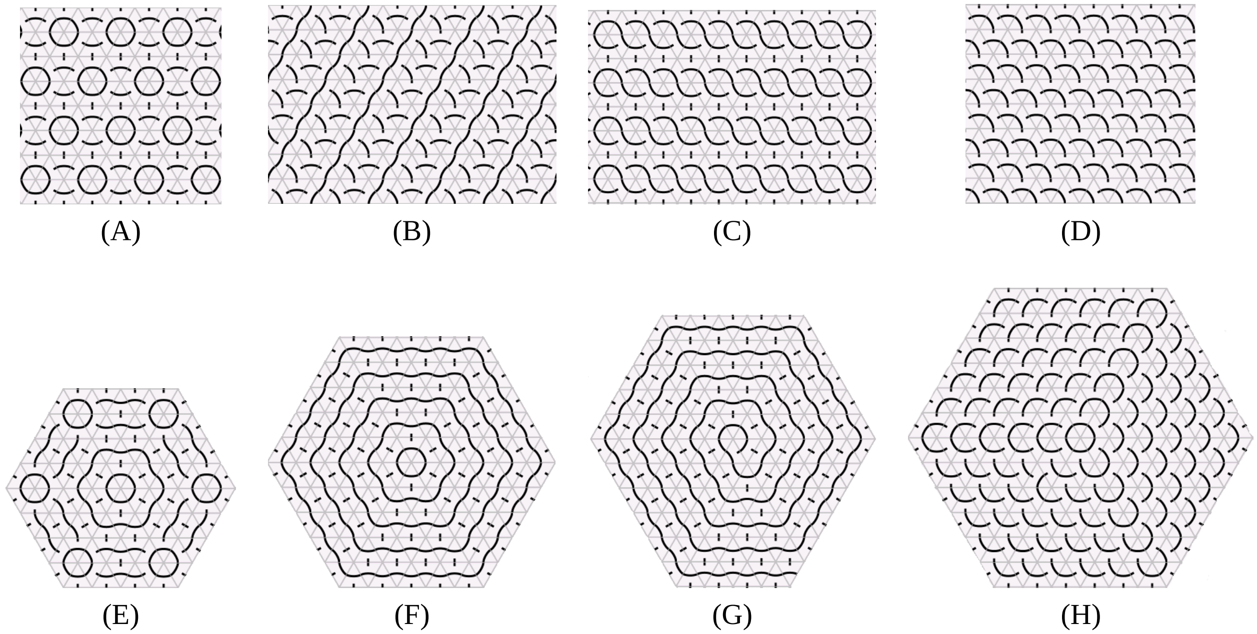

Figure 3: *Eight patterns possible using the simple motif show in Figure 1. Patterns that can be extended infinitely far in all directions are possible, as shown in A through D. Patterns created on a hexagonal lattice with six fold (E and F) and three fold rotational symmetry (H) are also possible. Space-filling curves, such as spirals (G), can be easily created.*

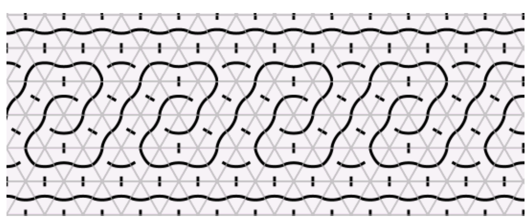

Figure 4: *An example of a frieze pattern obtainable from the motif in Figure 1.*

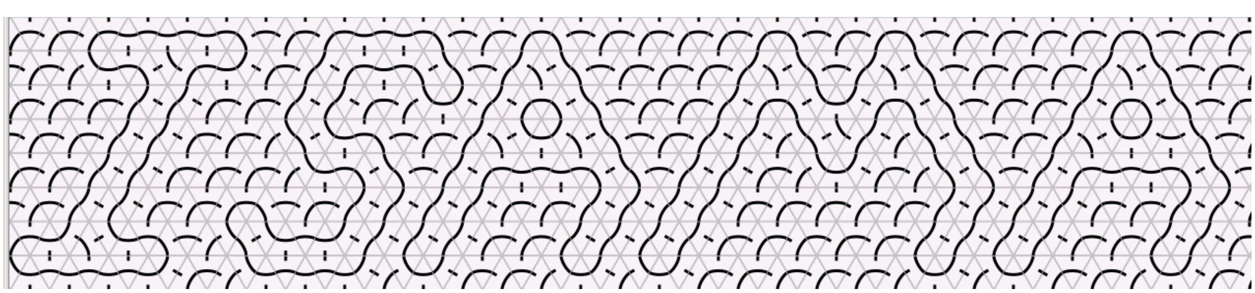

Figure 5: *An example showing how letters can be placed in a pattern comprised of modular triangular units decorated with the motif in Figure 1. The example text reads ISAMA.*