Patterns from Archimedean Tilings Using Generalized Truchet Tiles Decorated with Simple Bézier Curves

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Abstract

Decorated tiles with simple motifs have been used to enhance the visual appeal of tilings by infusing the underlying tessellation with additional patterns. This paper explores a generalization of Truchet tiles by decorating tiles made from regular polygons with simple Bézier curves and considering more than one arc per side. Examples of the generalized tilings for each of the Archimedean tilings are presented using one and two arcs per side. The tension present between the global irregularity and both the local similarity and positional regularity of the generated curves provides excitement and movement not present in the underlying tessellations.

1 Introduction

Historically it has been known that only three regular polygons allow a monohedral tiling of the plane, namely the square, triangle, and hexagon (the regular tessellations). When considering vertex types, there are an additional eight other possible tessellations of the plane involving combinations of regular polygons (the semiregular tessellations). The collection of regular and semiregular tessellations are commonly called Archimedean tilings [1].

Decorated tiles with simple motifs have been used to enhance the visual appeal of tilings by creating additional patterns. The commonly known Truchet tiles, square tiles decorated with quarter circle arcs, date to paper by Smith [2]. A more complete historical account of these tiles is given by Reimann [3] and Browne [4]. Browne recently presented an extension of Truchet tiles to triangles and hexagons [4].

2 Methods

This paper explores a generalization of Truchet tiles to include all regular polygons and using multiple arcs per side. In extending the concept of Truchet tiles, it is instructive to consider the essential features that make Truchet tiles appealing. One reason for the appeal is the arcs of adjacent tiles are not only continuous, but also have a continuous first derivative resulting in a visually smooth transition regardless of tile orientation. In addition, the meandering paths created are roughly equally spaced, providing a relatively uniform filling of the plane.

Generalizing the Truchet tile concept to decorate other polygons with an even number of sides is straightforward. One can connect pairs of sides with a simple arc. The arc between sides could simply be a segment from a circular arc or a more complex curve such that the derivative of the curve is perpendicular to the side of the polygon at each endpoint. Parallel sides are connected using a straight line segment.

When the polygon has an odd number of sides, there will alway be an unpaired side which must be treated as a special case. One solution is to have two arcs meet on one side [4], which results in a bifurcating structure rather than a set of closed meandering paths (loops). The method presented here uses a short terminal line segment that is perpendicular to the polygon side. This results in patterns that are comprised of both loops and unclosed meandering paths.
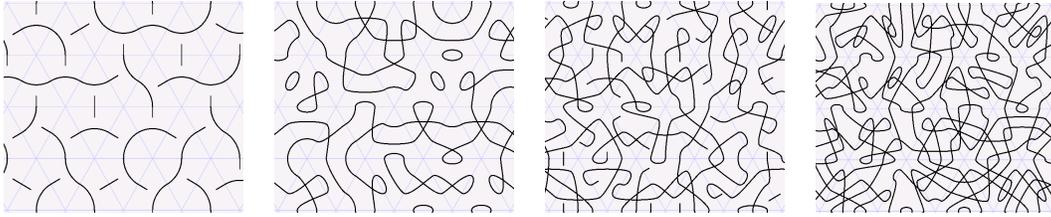
Figure 1: *Examples illustrating the arced motifs randomly applied to the 3,3,3,3,3,3 tiling of the plane using 1, 2, 3 and 4 arc endpoints per side (left to right).*

The other generalization presented here involves subdividing the sides of the polygon into multiple equal length segments. Making the subdivisions equal in length allows arcs from adjacent tiles to form continuous segments. In polygons with an even number of sides and connecting arcs from parallel sides, one may need to use a Bézier curve [5] with an inflection point to provide the proper smoothness.

3   Results and Discussion

Figure 1 shows examples of the decorated tilings described above for the 3,3,3,3,3,3 tiling using up to four arcs per side. Figure 2 shows examples of the decorated tilings described above for each of the Archimedean tilings using one and two arcs per side. The perpendicular distance of the control points from the polygon sides was chosen to provide roughly circular arcs in triangular and square tilings. The control point distance was increased as the number of sides increased, resulting in varying amounts of local curvature in the motifs.

In general the patterns produced are similar to and more complex than the conventional Truchet tiling. When one arc per polygon side is used and the underlying tessellation contains a triangle, the resulting patterns contain loops as well as unclosed meandering paths. While the curves in the tilings are fairly unique, they contain visually similar elements due to their underlying construction. The curves become increasingly varied as the number of different polygons and the number of sides of the underlying polygons increase. The tension present between the both local similarity and positional regularity and the global irregularity of the generated curves provides excitement and movement not present in the underlying tessellations.

The concept presented here is well-suited to applications where a large field is to be tiled, such as in architecture or textiles. While shown here as black curves on a white background, it is also possible to color each curve a unique color.

Future work includes using these decorated tiles to decorate polyhedra and other tilings. A more detailed investigation of the Bézier control points is warranted including the distance from the polygon sides and the constraining the distances to be constant in any given polygon.

References

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3,3,3,3,3,3

4,4,4,4

6,6,6

3,3,3,4,4

3,6,3,6
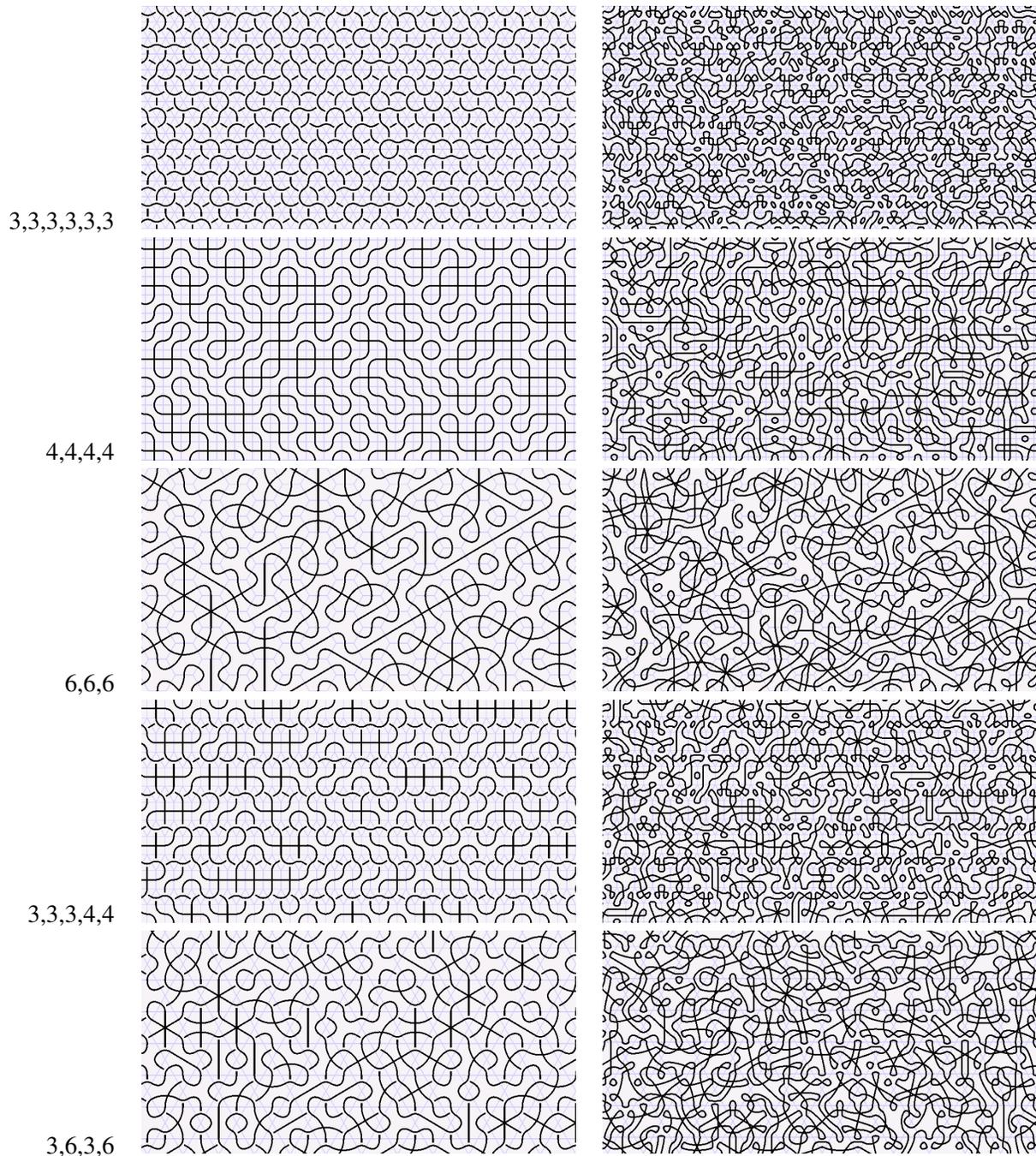
Figure 2: *Examples illustrating the arced motifs randomly applied to each of the eleven basic Archimedean tilings of the plane. The underlying tiling is also shown. In the left column, the midpoint of each side is connected to the midpoint of another side in the polygon using a Bézier curve; this is repeated for each side of the polygons in the tiling. In the case where there is an odd number of sides (the triangle), a terminal short line segment is used on one of the sides, resulting in many relatively short disjoint arcs. When each polygon comprising the tiling has an even number of sides, such as 4,12,6, the arcs are long and meandering. In the right column, each side is trisected and the two points per side of each polygon are connected with Bézier curves. Note the need for motifs with an inflection point in the polygons with an even number of sides. Continued on next page.*

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3,3,3,3,6

4,8,8

3,3,4,3,4

3,4,6,4

4,12,6

3,12,12
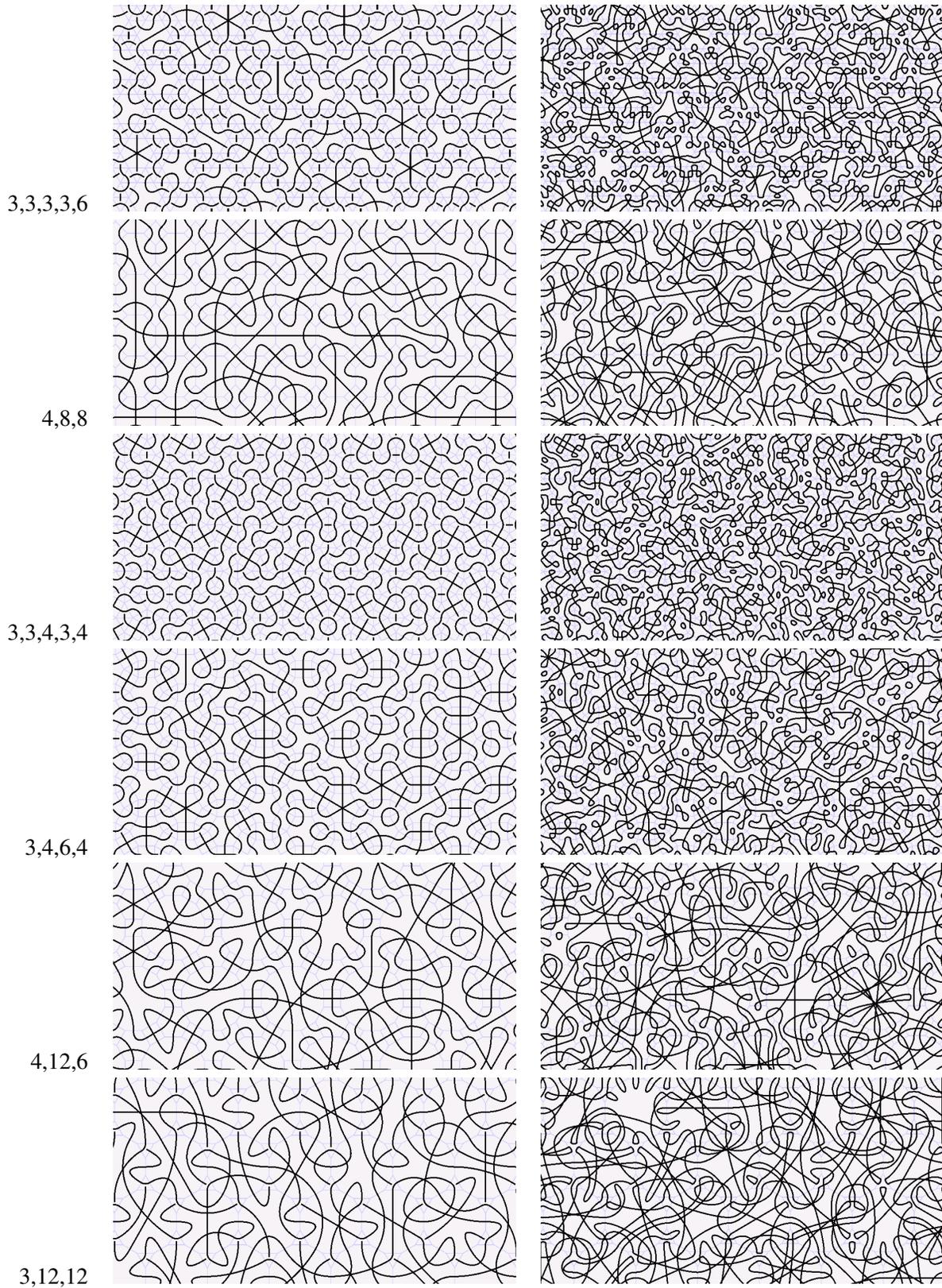
Figure 2: *Continued from previous page.*